B.sc(H) part 1 paper 1

Topic:problems based on inverse of the matrix

Subject:mathematics

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Ex 1. Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 6 & 7 & 9 \end{bmatrix}$.

Soln. Here $|A| = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 6 & 7 & 9 \end{vmatrix}$

$= 1(36 - 35) - 2(27 - 30) + 3(21 - 24)$

$= 1 + 6 - 9 = -2 \ (\neq 0)$.

Since $|A| \neq 0$, $\therefore$ A is non-singular and hence A^{-1} exists.

Now, the cofactors of the elements of the first row of $|A|$ are

$\begin{vmatrix} 4 & 5 \\ 7 & 9 \end{vmatrix}, -\begin{vmatrix} 3 & 5 \\ 6 & 9 \end{vmatrix}, \begin{vmatrix} 3 & 4 \\ 6 & 7 \end{vmatrix}$ respectively i.e. 1, 3, –3.

Again the cofactors of the elements of the second row of $|A|$ are

$-\begin{vmatrix} 2 & 3 \\ 7 & 9 \end{vmatrix}, \begin{vmatrix} 1 & 3 \\ 6 & 9 \end{vmatrix}, -\begin{vmatrix} 1 & 2 \\ 6 & 7 \end{vmatrix}$ respectively i.e. 3, 9, –5.

Again the cofactors of the elements of the third row of $|A|$ are

$\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix}, -\begin{vmatrix} 1 & 3 \\ 3 & 5 \end{vmatrix}, \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$ respectively i.e. –2, 4, –2.

Hence the matrix B, whose elements are the cofactors of the corresponding elements of $|A|$ is

$$B = \begin{bmatrix} 1 & 3 & -3 \\ 3 & -9 & 5 \\ -2 & 4 & -2 \end{bmatrix}.$$

Hence adj A = transpose of $B = \begin{bmatrix} 1 & 3 & -2 \\ 3 & -9 & 4 \\ -3 & 5 & -2 \end{bmatrix}$.

$$\therefore \quad A^{-1} = \frac{1}{|A|} \text{adj. } A = \frac{1}{-2} \text{adj} A = \begin{bmatrix} -\frac{1}{2} & -\frac{3}{2} & 1 \\ -\frac{3}{2} & \frac{9}{2} & -2 \\ \frac{3}{2} & -\frac{5}{2} & 1 \end{bmatrix}.$$

Ex 2 **Find the inverse of the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$.**

Soln. We have, $|A| = \begin{vmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{vmatrix}$

$$= 0(2-3) - 1(1-9) + 2(1-6)$$

$$= 0 + 8 - 10 = -2 \ (\neq 0).$$

Since $|A| \neq 0$, $\therefore$ A is non-singular and hence A^{-1} exists.

Now, the cofactors of the elements of the first row of $|A|$ are

$\begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix}, -\begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix}$ respectively i.e. –1, 8, –5.

The cofactors of the elements of the second row of $|A|$ are

$-\begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 0 & 2 \\ 3 & 1 \end{vmatrix}, -\begin{vmatrix} 0 & 1 \\ 3 & 1 \end{vmatrix}$ respectively i.e. 1, −6, 3.

The cofactors of the elements of the third row of $|A|$ are

$\begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix}, -\begin{vmatrix} 0 & 2 \\ 1 & 3 \end{vmatrix}, \begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix}$ respectively i.e. −1, 2, −1.

Therefore the matrix B whose elements are the cofactors of the elements of $|A|$ is

$$B = \begin{bmatrix} -1 & 8 & -5 \\ 1 & -6 & 3 \\ -1 & 2 & -1 \end{bmatrix}.$$

Hence adj A = transpose of $B = \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$.

Now, $A^{-1} = \dfrac{1}{|A|}$ adj. A

$$\therefore \quad A^{-1} = -\frac{1}{2}\begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 1/2 & -1/2 & 1/2 \\ -4 & 3 & -1 \\ 5/2 & -3/2 & 1/2 \end{bmatrix}.$$

Ex3. **Find the adjoint and inverse of the matrix**

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$ **[Haz. 1996H; R.U. 1973H, 92H]**

Soln. Let the matrix be denoted by A.

$$\text{Then } |A| = \begin{vmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix};$$

$$= \cos^2\theta + \sin^2\theta = 1.$$

$\therefore$ A is non-singular.

The matrix B whose elements are the cofactors of the corresponding elements of $|A|$ is

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \text{ adjoint of } A = B' = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \text{ The inverse of } A = A^{-1} = \frac{1}{|A|}[''\,]$$

$$= \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Ex 4 **If the product of two non-zero matrices is a zero matrix, show that both of them must be singular matrices.**

Soln. Let each of the two matrices A and B, be a non-zero matrix of $n \times n$ order. Given $AB = O$.

It is to prove that $|A| = 0$ and $|B| = 0$.

Let $|B| \neq 0$. Then B^{-1} exists.

Hence from the given equation $AB = O$, we get

$$AB\,B^{-1} = O \Rightarrow AI = O \Rightarrow A = O.$$

But A is not a zero matrix, therefore $|B|$ is necessarily $= 0$.

Again, let $|A| \neq 0$, then A^{-1} exists.

Hence from the equation $AB = O$, we get

$$A^{-1}AB = O \Rightarrow IB = O \Rightarrow B = O.$$

But B is not a zero or null matrix.

Therefore, necessarily, $|A| = 0$.

Thus both $|A|$ and $|B| = 0$.

This means that both the matrices A and B are singular.